A Hierarchical mixture model for sparse broadband time varying acoustic response functions with application to reception of M-ary orthogonal spread spectrum communications at very low SNR

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Outline:

1. Introduction: Very low SNR communications
   • M-ary orthogonal spread spectrum communications for the underwater acoustic channel.
   • Underwater acoustic response functions, their sparsity and angle-Doppler-delay spreading.

2. Channel modeling: Mixture Gaussian Model (mGm)
   • mGm over frequency-Doppler-beam for acoustic response function estimation.
   • Time varying dilation estimation from posterior estimates of sparse acoustic response.
   • Iterative reception at very low SNR with experimental results

3. Conclusions / Future work
4. Overview of the State of Art
M-ary orthogonal spread spectrum acoustic communications through shallow water environments

\[ \hat{s} = \arg \max Pr[s | r] = s \]

\[ \arg \max \int Pr[s | r, h] \cdot p(h | r)dh \]

Transmitter

\[ \ldots 0101100100 \ldots \]

channel estimation

decisions (soft/hard)

dilation compensation

\[ \hat{s} = \arg \max Pr[s | r] = s \]

\[ \arg \max \int Pr[s | r, h] \cdot p(h | r)dh \]

Receiver

\[ \ldots 0101100100 \ldots \]
Advantages of M-ary orthogonal spread spectrum signaling:

1. Asymptotically capacity achieving for AWGN channel error rate performance. ~ suitable for very low SNR communications.
3. Amenable to MIMO paradigm:

   \[ R = K \times \log_2 \frac{M}{K} > \log_2 M, \quad K = \# \text{channels} \]

More importantly with L-band orthogonality:

\[ Wr \mid S, b, h, \Sigma, \sim N(Wr \mid S_b h, I_N), \quad Wr \mid S, h, \Sigma \sim M^{-K} \sum_b N(Wr \mid S_b h, I_N). \]

\[ x = \sum_b S_b Wr \mid h, \Sigma \quad E[x] = h \quad S \in M – \text{ary}, L – \text{orthogonality}. \]

4. Broadband channel response can be coherently estimated without knowledge of the symbols:

   at a penalty of \( 10 \log M \) dB compared to exact knowledge of symbols.

   ... Processing gain can accommodate this loss.
Signals $\{M-N-L\}$ form an M-ary set with processing gain $N$ and L-band orthogonality, if $S_a \in C^{N \times L}$ the convolution operator associ. with $s_a$:

$$\forall s_a, s_b \in \{M-N-L\} :$$

$$S'_a \ast S_b = 1_{a=b} \cdot (I_{L \times L} + R_{a,b}) + 11_{a \neq b} \cdot E_{a,b}$$

$$\text{diag}(R_{a,b}) = 0, \quad |R_{a,b}(i, j)| < 1/N,$$

$$|E_{a,b}(i, j)| < 1/N$$
A hierarchical model for sparse dynamic shallow water acoustic response functions

$$\pi_{vfk} \sim \text{Beta} (\alpha, \beta_g) \quad , \quad E[\pi_{vfk}] = \pi_k \cdot D_{v-gf}$$

$$z_{vfk} \sim \text{Ber}(\pi_{vfk}),$$

$$h_{vfk} \mid z_{vfk}, \lambda_{fk}, \varepsilon_k \sim N_{h_{vfk}} \left( 0, \lambda_{fk}^2 \right) \cdot N_{h_{vfk}}^{1-z_{vfk}} \left( 0, \varepsilon_k^2 \right)$$

- The proportions $\pi_{k,vf}$ indicate the prior degree of sparsity for the $k$-th beam.
- The indicator variables $z_{k,vf}$ specify which frequency/Doppler/beams are ensonified with coherent energy.
- The Gaussian variates $h_{k,vf}$ are the amplitude and phase of the response function at each frequency/Doppler/beam.
Validation of mixture Gaussian model (mGm) for sparse acoustic response

\[ h_{\Delta k} \mid \pi_k \lambda_{jk} \epsilon_k \sim \pi_k N_{h_{\Delta k}}(0, \lambda_{jk}) + (1 - \pi_k) N_{h_{\Delta k}}(0, \epsilon_k) \]

Gendron, JASA EL 2017
Posterior conditional density of acoustic response function is MG, all expectations are easily handled:

\[
p(h_{vfk} | r, s, \pi, \lambda, \varepsilon) = \pi_{vfk}(r) \cdot N(h_{vfk} | \gamma \lambda h^{LS}, \gamma \lambda) + (1 - \pi_{vfk}(r)) \cdot N(h_{vfk} | \gamma \varepsilon h^{LS}, \gamma \varepsilon)
\]

\[
\pi_{vfk}(r) = \frac{\pi_{vk} N(h^{LS}_{vfk} | 0, \lambda_f + 1)}{\pi_{vk} N(h^{LS}_{vfk} | 0, \lambda_f + 1) + (1 - \pi_{vk}) N(h^{LS}_{vfk} | 0, (\varepsilon_k + 1))}
\]

Posterior probability that the beam-Doppler Freq. coefficient is coherently ensonified

\[
\gamma_x = \frac{x^2}{x^2 + 1}
\]
Posterior conditional density of whitened acoustic response function is also MG so that all expectations are easily handled:

\[ p(h_{vfk} \mid s, r) \propto N(h_{vfk} \mid h_{vfk}^{LS}, I) \cdot p(\overline{h} \mid \pi, \lambda, \varepsilon), \]

\[ h^{LS} = U_{vfk} S^W \overline{r} \]

\[ p(h_{vfk} \mid r, s, \pi, \lambda, \varepsilon) = \pi_{vfk}(r) \cdot N(h_{vfk} \mid \gamma_{\lambda} h^{LS}, \gamma_{\lambda}) + (1 - \pi_{vfk}(r)) N(h_{vfk} \mid \gamma_{\varepsilon} h^{LS}, \gamma_{\varepsilon}) \]

\[ \gamma_x = \frac{x^2}{x^2 + 1} \]

\[ \pi_{vfk}(r) = \frac{\pi_v N(h^{LS}_{vfk} \mid 0, \lambda_f + 1)}{\pi_v N(h^{LS}_{vfk} \mid 0, \lambda_f + 1) + (1 - \pi_v) N(h^{LS}_{vfk} \mid 0, (\varepsilon_k + 1))} \]

Posterior probability that the coefficient is coherently ensonified
Fast point estimation with posterior mean:

\[ E[h_{vfk} \mid r, s, \pi_0, \lambda, \epsilon] = (\pi_r \gamma_{\lambda} + (1-\pi_r)\gamma_{\epsilon}) \cdot h_{vfk}^{LS} \]

The posterior mean is a shrinkage operator applied to LS solution ~ strong function of data, large coefficients unchanged.

\[ \text{var}[h_{vfk} \mid r, s, \lambda, \epsilon] = \pi_r \gamma_{\lambda} + (1-\pi_r)\gamma_{\epsilon} + \pi_r (1-\pi_r)(\gamma_{\lambda} - \gamma_{\epsilon}) \left|h_{vfk}^{LS}\right|^2 \]

Variance is average of conditional variances of mixture models + variance assoc. with ambiguity between models.
Time invariant dilation estimation:

Use posterior mean $E \left[ h_{\Delta f_k} \mid r, S_b \right]$ of acoustic response as function of Doppler-frequency to estimate $\partial \tau$.

$$\partial \hat{\tau} = \arg \max_{\partial \tau} \text{Re} \left( h_{\Delta f_k}(\partial \tau), \hat{h}_{\Delta f_k} \right)$$

estimate is a simple linear regression maximizing channel energy about hypothesized Doppler-freq.axis.

$$\partial \hat{\tau} \approx \int_{B \Delta} \frac{v}{(f + f_c)} \left| \hat{h}_{vf_k} \right|^2 df dv$$
Low SNR reception:

\[ \hat{s} = \arg \max_s \int p(h \mid s, r) \cdot \Pr[s \mid r] dh \]

Soft - : \( \Pr[s = s_l \mid r] = \pi_s^{(k)} (l \mid r, h = \hat{h}) \)

Hard - : \( s^{(k)} = \arg \max_s p(r \mid s, h = \hat{h}) \)

Soft - \( \hat{h} = E[h \mid r] = \sum_l \pi_s (l) E[h \mid r, s = s_l] \)

Hard - : \( \hat{h}^{(k)} = E[h \mid r, s = s^{(k-1)}, \theta = \theta^{(k-1)}] \)

\( \theta^{(k)} (r) = \text{Empirical Bayes Point Estimators} \)
At-sea tests

St Margaret’s Bay

SIMO configurations at 11 kHz. Downward refracting ssp, soft lossy bottom, 70m water depth. Fixed-fixed, drifting and mobile source receiver scenarios. (Teledyne Benthos, Dale Green)

Buzzard’s Bay

Broadband (25kHz) at a center frequency of 35 kHz. Drifting source-receiver scenarios. (WHOI, Lee Freitag)

<table>
<thead>
<tr>
<th>Environment</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source depth</td>
<td>7.5m</td>
</tr>
<tr>
<td>Water depth</td>
<td>14-16m</td>
</tr>
<tr>
<td>ssp</td>
<td>iso-velocity</td>
</tr>
<tr>
<td>Bathymetry</td>
<td>rough (~1ft)</td>
</tr>
<tr>
<td>Bottom sediment</td>
<td>sandy/rocky</td>
</tr>
</tbody>
</table>
St. Margaret’s Bay, coh. v noncoh. , Range >3km, fc = 10 kHz, BW=5kHz
Buzzard’s Bay: 8-ary fc=32.5kHz, B=25kHz

1 km

2 km

220700-RB2-T15-01, 8-ary orth., MFC and WPC w/ non-coh. det.

rSNR (dB)

131 bits/sec

P_b

SNR/bit/element (dB)

10^{-3}

10^{-4}

10^{-10}

4 6 8 10 12

AWGN, non-coh det., 1 ele.
MFC: L=8, 1 ele.
MFC: L=8, 2 ele.
MPC: L=8, 3 ele.
WPC: w' 1 ele.
WPC: w' 2 ele.
WPC: w' 3 ele.
Conclusions:

1. Coherent estimation of the acoustic response can be effective even at very low SNR with large spreading gain M-ary orthogonal signaling.
2. A mixture Gaussian model provides a computationally fast statistically efficient MMSE estimators of the sparse acoustic response.
3. Both time invariant and time varying Doppler estimation and compensation can be accomplished from these posterior estimates of the acoustic response function.

Future work:

1. Extend M-ary orthogonal signaling scheme to MIMO systems.
2. Incorporate “persistence” (dependence in indicator variables) into mixture-Gaussian model.
3. Apply to basin scale long range sound duct channels.
Very low SNR communications:

1. M-ary orthogonal spread spectrum signaling:
   - Benjamin Sherlock et al., Signal and receiver design for low power acoustic communications using M-ary orthogonal communications, OCEANS 2015, Genova.
   - A receiver structure for coherent reception of M-ary orthogonal spread spectrum acoustic communications at very low SNR in shallow water environments, P. Gendron, OCEANS 2013,

2. OFDM for low SNR/low rate communications:

3. FH-FSK schemes:

4. Direct-Sequence Spread Spectrum:
Modeling the sparsity of the multipath response

1. Hierarchical models and Empirical Bayesian methods:

2. RLS/DFE sparse algorithm

3. Subspace methods and adaptive basis search:
   - Sparse channel estimation for Multicarrier Underwater Acoustic Communications: Subspace methods to Compressed Sensing, C. Berger et al., *IEEE Trans. on Signal Processing*, Vol. 58, No. 3, March 2010.(Applies orthogonal matching pursuit (OMP) and Basis Pursuit (BP) to sparse channel estimation for OFDM)